On Pricing Schemes in Data Center Network with Game Theoretic Approach

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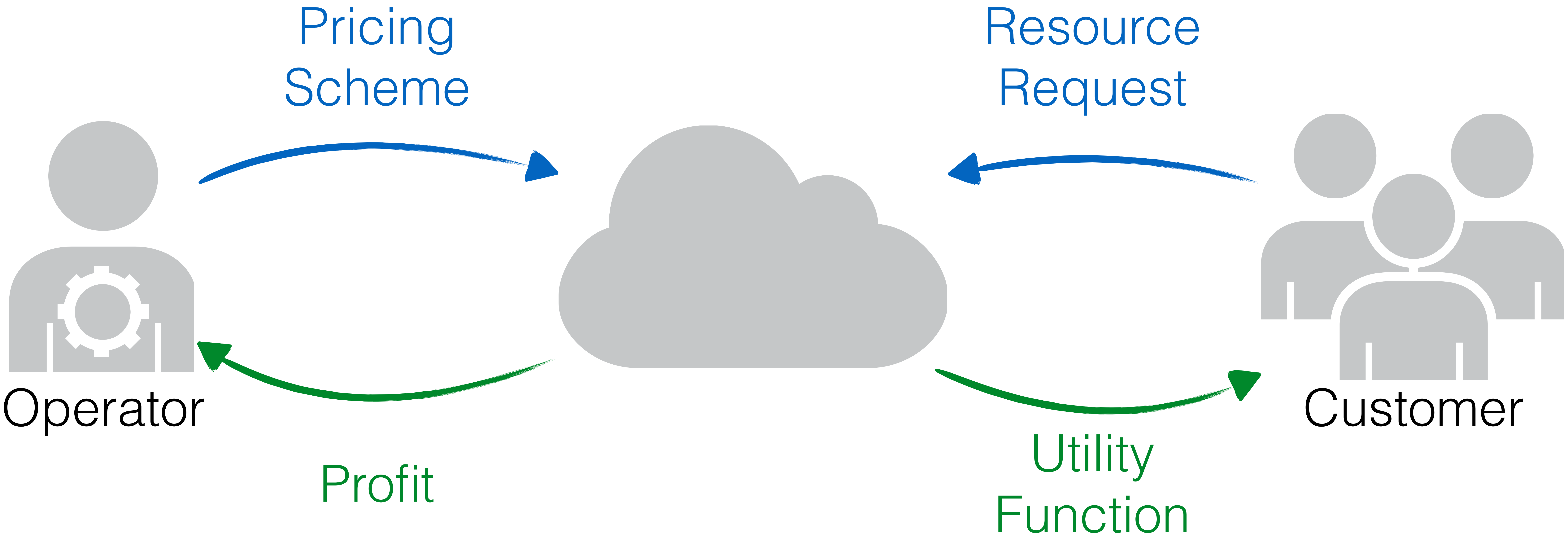
Operator and Customer



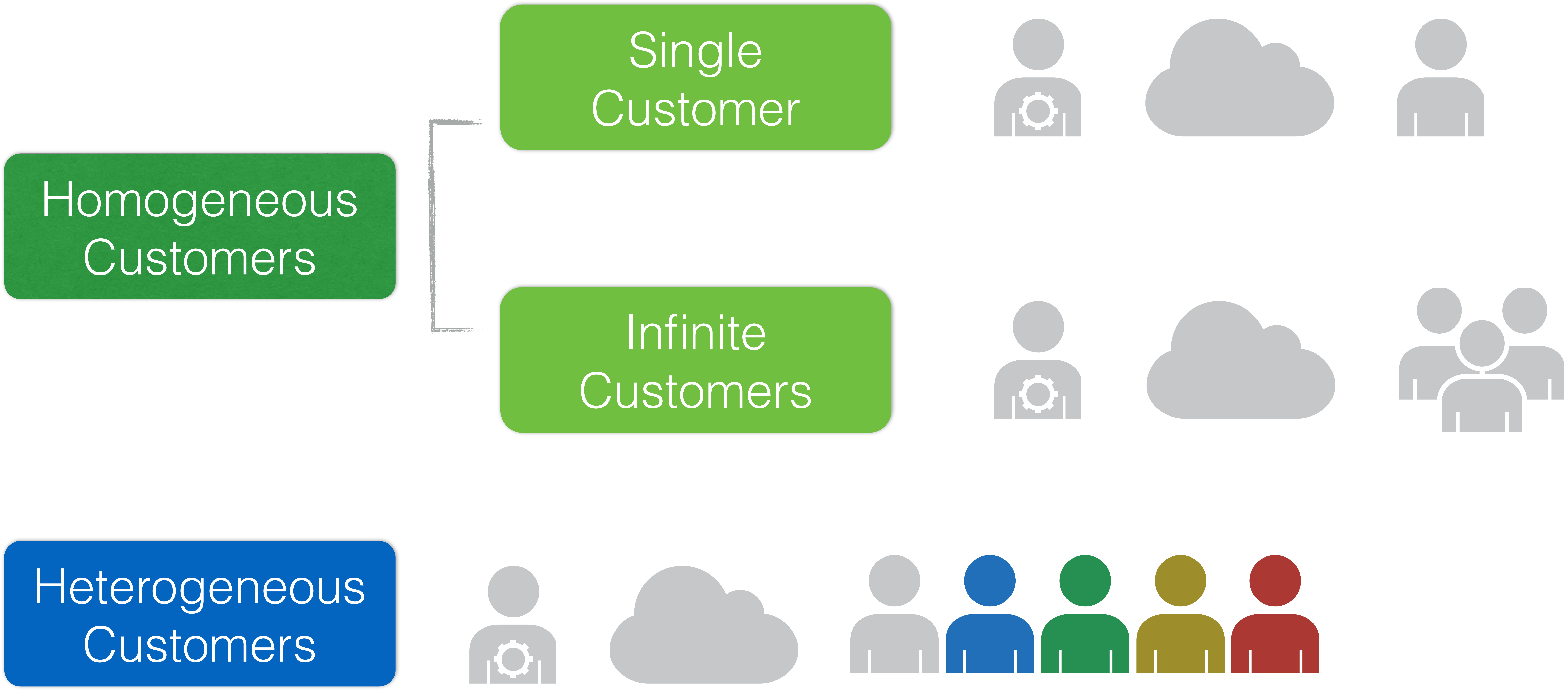
Pricing in DCN

Problem:

- The *operator* announces a price for the resources
- Given this price, the *customer* selfishly determines how many resources it requests

Stackelberg equilibrium will lead to a Pareto-inefficient outcome

Modelling the DCN Operator and Customers



Single Customer

- Customer's utility: $U(d,p) = f(d) - pd$
- Operator's utility: $V(d,p) = pd - vg(d)$

To maximize utility of the customer: $d = f'^{-1}(p) = h(p)$

$$V(d(p),p) = ph(p) - vg(h(p))$$

Therefore, operator can maximize its utility by solving:

$$\frac{dV(d(p),p)}{dp} = h(p) + ph'(p) - vg'(h(p))h'(p) = 0$$

Single Customer

1) Does Stackelberg equilibrium always exist?

2) Is the outcome at Stackelberg equilibrium Pareto-efficient?

3) If 2) is not the case, how to get a Pareto-efficient solution?

Single Customer

Example 1

- set $f(d) = \ln d, \; g(d) = d^{\alpha}$

$$\frac{dV(d(p),p)}{dp} = \frac{1}{p} + p(-\frac{1}{p^2}) + vap^{-a-1} = vap^{-a-1} > 0$$

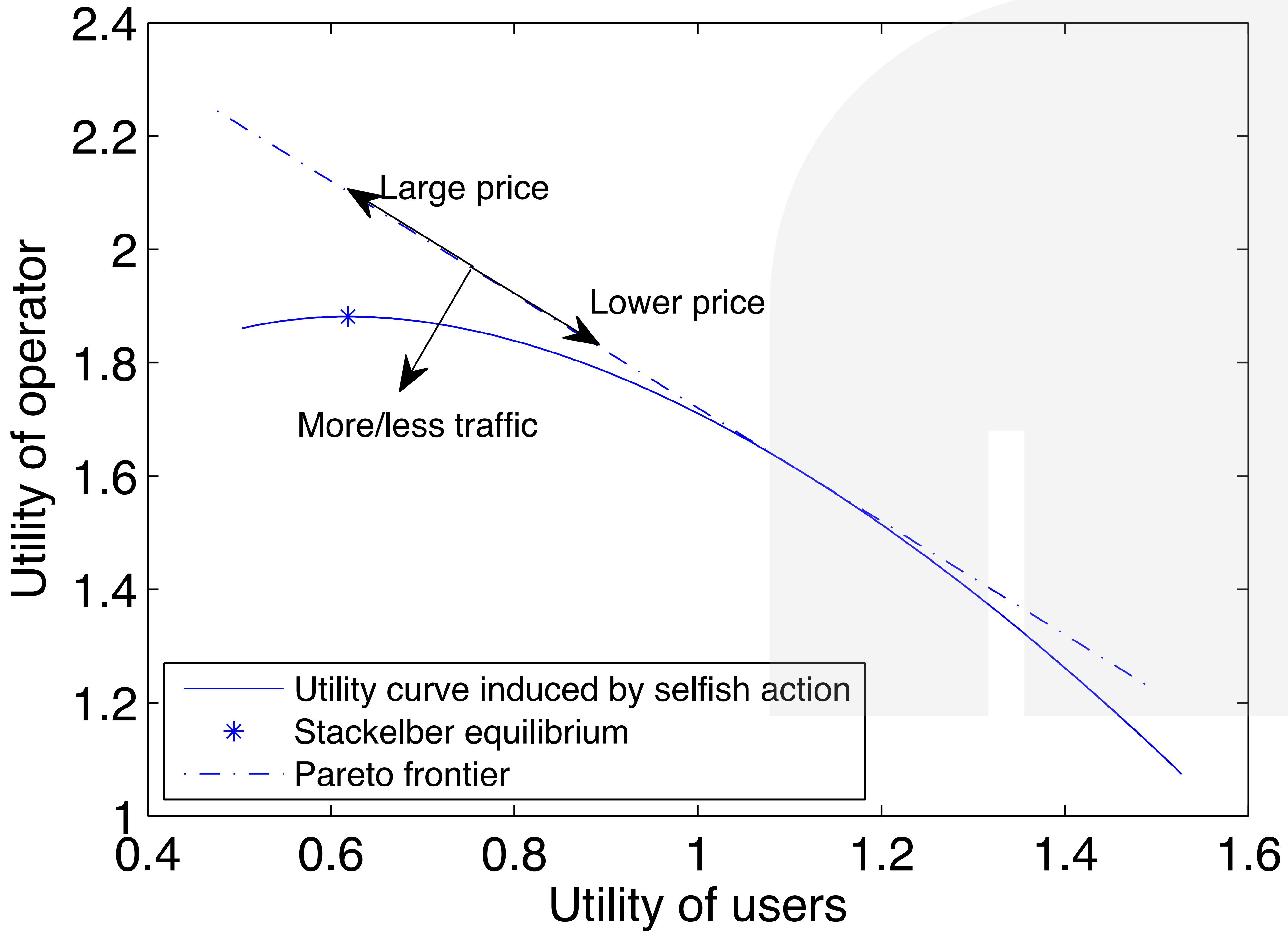
- Stackelberg equilibrium always does not exist

Example 2

- set $f(d) = M - Me^{-d}$

operator will choose a price p such that

$$\ln\frac{p}{M} + 1 = v\alpha\frac{1}{p}(-\ln\frac{p}{M})^{\alpha-1}$$



Single Customer

Theorem 1

Assume the demand quantity sent by customer at Stackelberg equilibrium and Pareto efficient solution are $d^{(s)}$ and $d^{(o)}$, respectively, then $d^{(s)} < d^{(o)}$.

Proof:

$$U(d^{(o)}, p^{(o)}) - U(d^{(s)}, p^{(s)})$$
$$= f(d^{(o)}) - f(d^{(s)}) - [f'(d^{(o)})d^{(o)} - f'(d^{(s)})d^{(s)}]$$
$$< f'(\xi)[d^{(o)} - d^{(s)}] - f'(d^{(s)})[d^{(o)} - d^{(s)}]$$
$$= [f'(\xi) - f'(d^{(s)})][d^{(o)} - d^{(s)}] < 0 \quad \cdots\cdots\cdots \quad (1)$$

$$U(d^{(o)}, p^{(o)}) + V(d^{(o)}, p^{(o)}) \geq U(d^{(s)}, p^{(s)}) + V(d^{(s)}, p^{(s)}) \quad \cdots \quad (2)$$

$$V(p^{(o)}, d^{(o)}) \geq V(p^{(s)}, d^{(s)}) \quad \cdots\cdots \quad (3)$$

$$d^{(s)} = h(p^{(s)}) \leq h(p^{(o)}) = d^{(o)} \quad \cdots \quad (4)$$

Single Customer

The price for d units of resource: $B(d,p) = P + pd$ .... ❶

Theorem 2 Assume $T(d) = U(d,p) + V(d,p)$

$$\begin{cases} p = p^{(o)} \\ P = V(d^{(s)}, p^{(s)}) - V(d^{(o)}, p^{(o)}) + \frac{1}{2}(T(d^{(o)}) - T(d^{(s)})) \end{cases}$$

Pareto efficient and Stackelberg equilibrium

Proof: From ❶, $U_a(d,p) = f(d) - (P + p^{(o)}d)$

$$U_a(d^{(o)}, p^{(o)}) = f(d^{(o)}) - (P + p^{(o)}d^{(o)}) = U(d^{(o)}, p^{(o)}) - [V(d^{(s)}, p^{(s)}) - V(d^{(o)}, p^{(o)}) + \frac{1}{2}(T(d^{(o)}) - T(d^{(s)}))]$$

$$= \frac{1}{2}(T(d^{(o)}) + T(d^{(s)})) - V(d^{(s)}, p^{(s)}) > T(d^{(s)}) - V(d^{(s)}, p^{(s)}) = U(d^{(s)}, p^{(s)})$$

$$V_a(d^{(o)}, p^{(o)}) = (P + p^{(o)}d^{(o)}) - vg(d^{(o)}) = P + V(d^{(o)}, p^{(o)})$$

$$= V(d^{(s)}, p^{(s)}) + \frac{1}{2}(T(d^{(o)}) - T(d^{(s)})) > V(d^{(s)}, p^{(s)})$$

Infinite Customers

Lemma 1

- Customer's utility: $U(d,p) = f(d)/d - p$
- Operator's utility: $V(d,p) = pd - vg(d)$

When the operator announces a larger price, there will be less demand sent into the network.

Proof:

$$\left.\begin{aligned} f(d) &= pd \\ f'(d)d' &= d + pd' \end{aligned}\right\} \longrightarrow d' = \frac{d^2}{f'(d)d - f(d)}$$

Since $f(0) = 0$,

$$f'(d)d - f(d) = f'(d)d - f(d) + f(0) = [f'(d) - f'(d^*)]d \qquad (d^* \in (0,d))$$

Thus, $d' < 0$ and d is a decreasing function of p

Infinite Customers

Lemma 2

The demand quantity d is a concave function of price of each unit resource, if and only if $f''(d)d' > 2$.

Proof:

$$f'(d)d' = d + pd'$$

↓ derivation

$$f''(d)(d')^2 + f'(d)d'' = 2d' + pd''$$

$$\left.\begin{array}{l} d'' = \dfrac{2d' - f''(d)(d')^2}{f'(d) - p} \\ f'(d) - p = d / d' \end{array}\right\} \longrightarrow \begin{array}{l} d'' = [2(d')^2 - f''(d)(d')^3] / d \\ \quad = [2 - f''(d)d'](d')^2 / d \end{array}$$

Infinite Customers

Lemma 3

$f''(d)d' > 2 \quad V\big(d(p),p\big)$ is a concave function of p .

Proof:

$f(d)$ is a concave function

$g(d)$ is a convex function

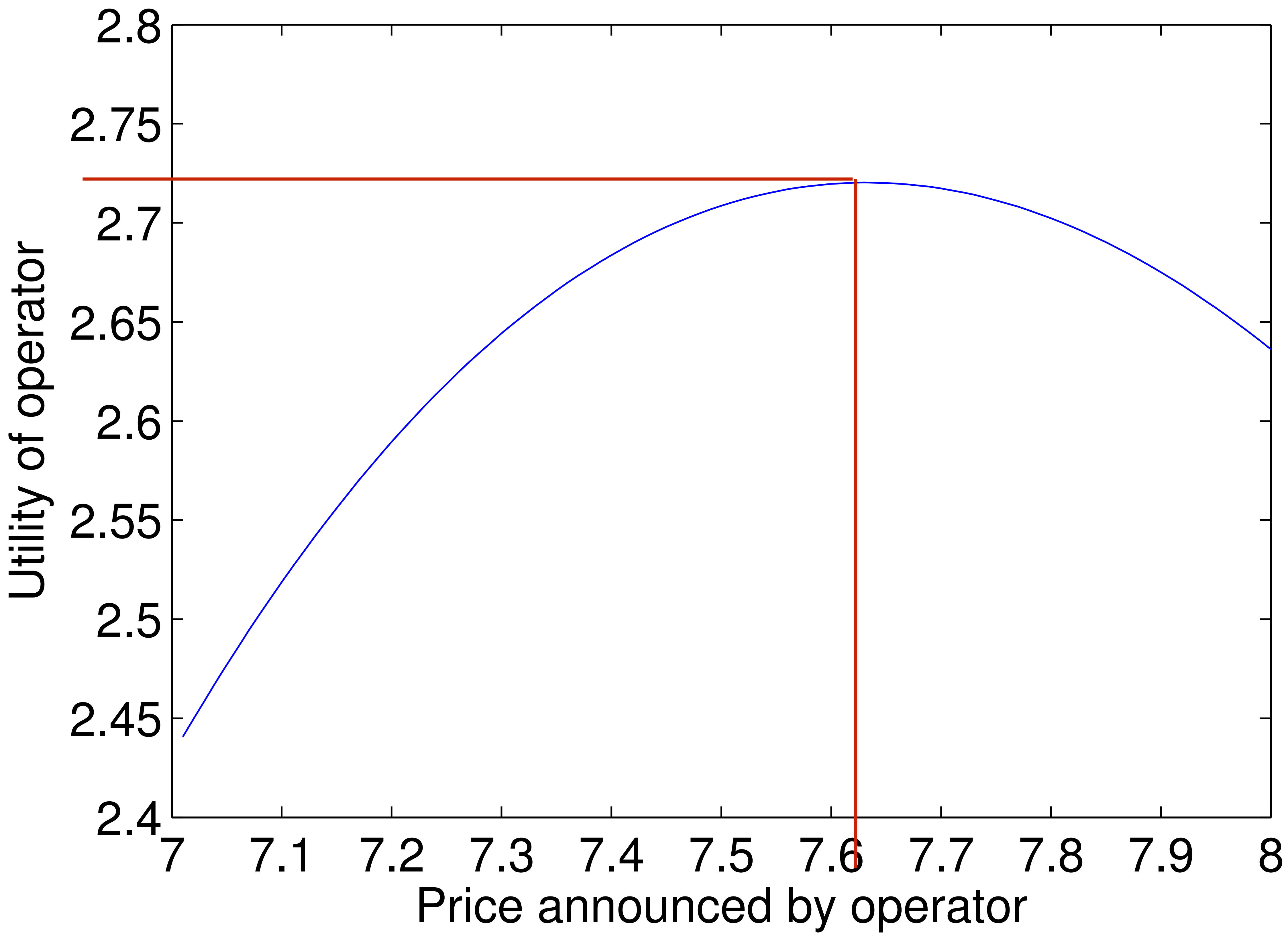
$V(d,p) = f(d) - vg(d)$ is a concave function of d

d is a decreasing and concave function of p.

Infinite Customers

Example 3 Utility of customers from one unit of resource:

- set $f(d) = M - Me^{-d}$



search such Stackelberg equilibrium if $V(d(p), p)$ is a concave function of p.

Heterogeneous Customer

- QoS requirement of a customer s
- marginal utility $u(s)$
- total demand quantity in the network is D
- network resource capacity is C
- distribution density function $q(s)$
- Customer's utility: $U(s,p) = D\int_{s}^{u^{-1}(p)} u(x)q(x)\,dx - pD\int_{s}^{u^{-1}(p)} q(x)\,dx$
- Operator's utility: $V(s,p) = p\int_{s}^{u^{-1}(p)} Dq(x)\,dx - vg(\int_{s}^{u^{-1}(p)} Dq(x)\,dx)$
- the QoS requirement constraint $D\int_{s}^{u^{-1}(p)} q(x)\,dx = sC$

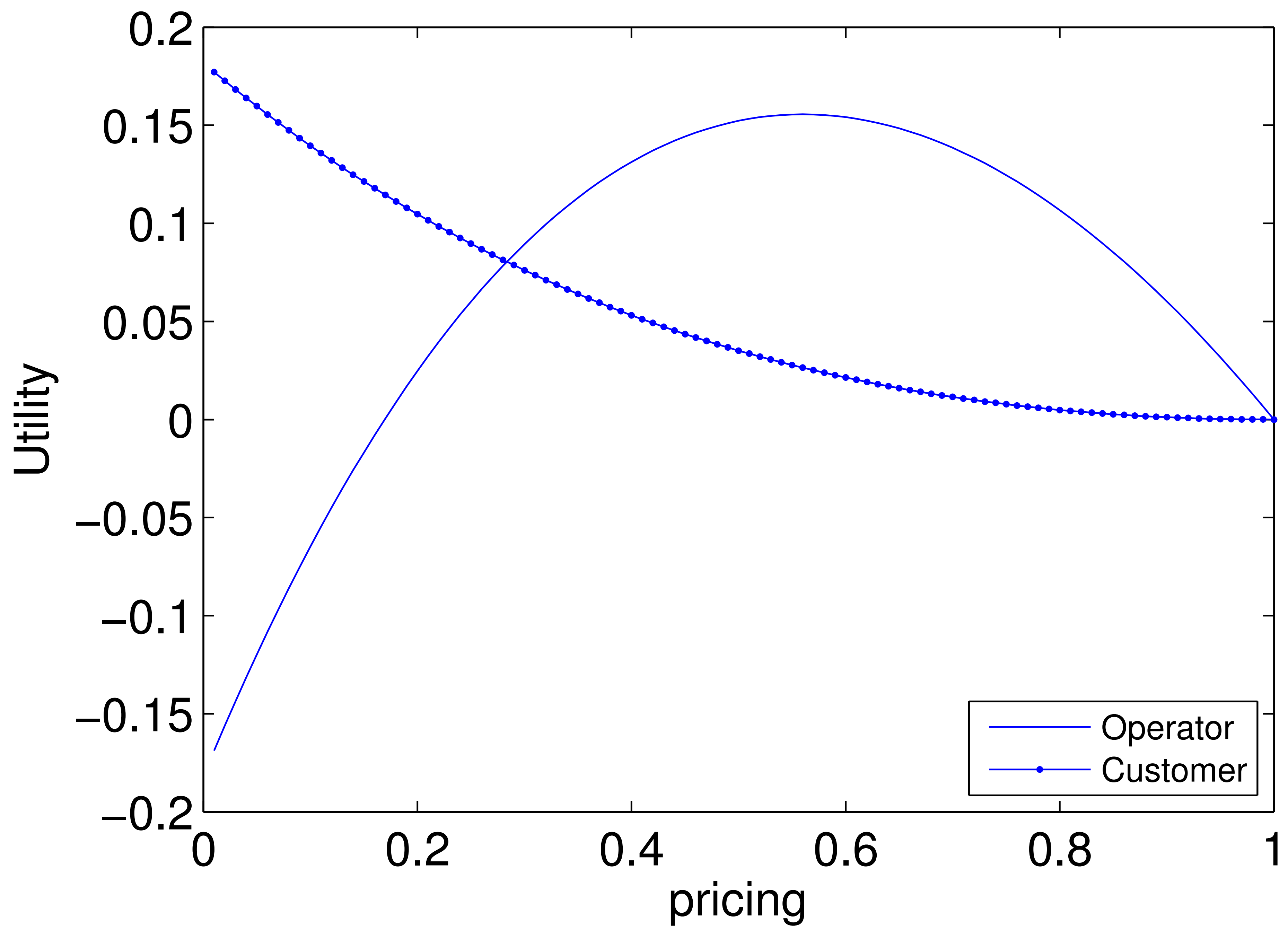
$$V(h(p),p) = p\int_{h(p)}^{u^{-1}(p)} Dq(x)\,dx - vg(\int_{h(p)}^{u^{-1}(p)} Dq(x)\,dx)$$

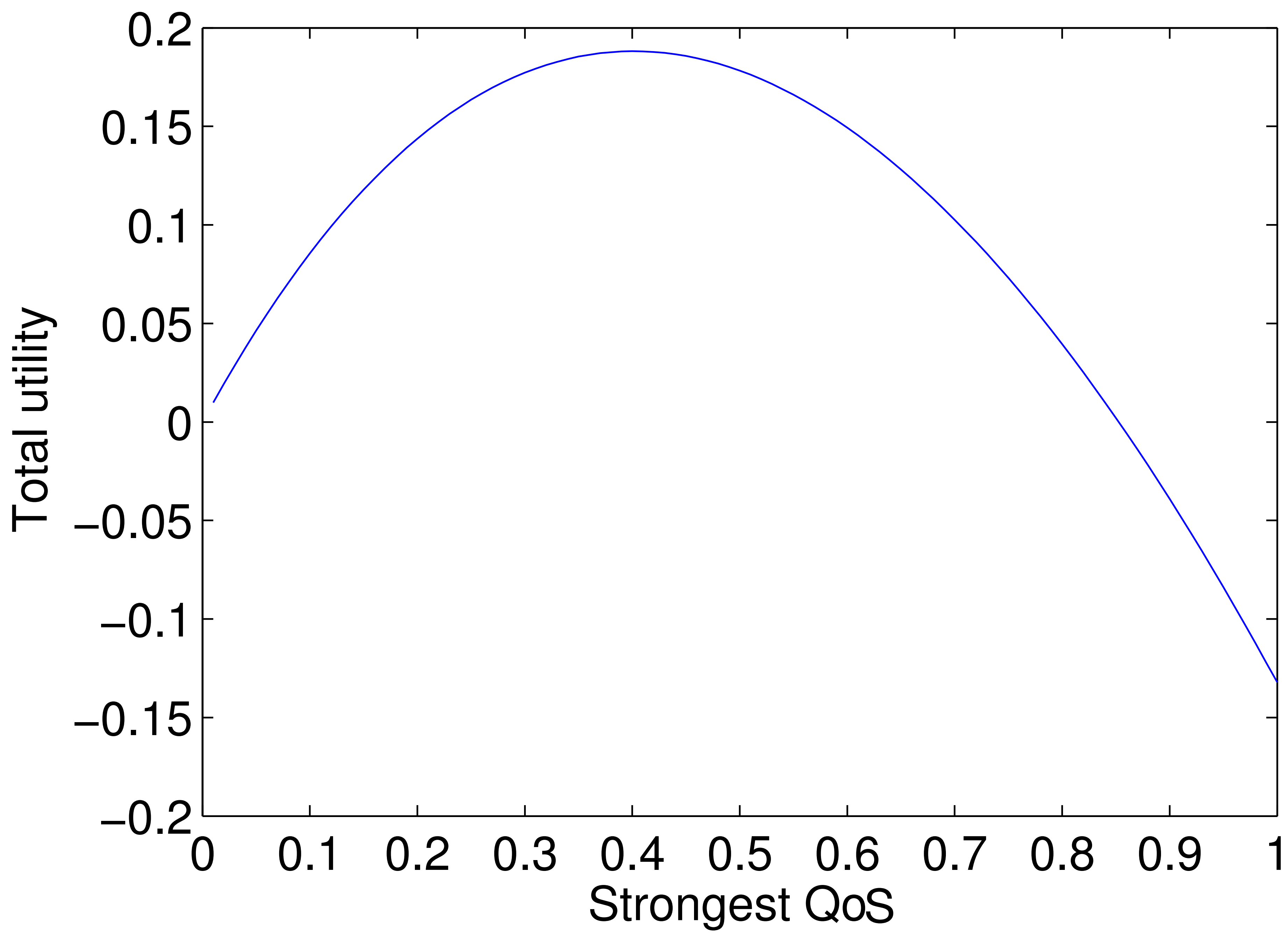
$$pD[I'(p)q(I(p)) - h'(p)q(h(p))] + D\int_{h(p)}^{I(p)} q(x)\,dx =$$

$$vDg'(D\int_{h(p)}^{I(p)} q(x)\,dx)[I'(p)q(I(p)) - q(h(p))h'(p)]$$

Heterogeneous Customer

Example 4 set $u(s)=e^{-s},\ q(s)=e^{-s},\ g(d)=d^2,\ D=2\ ,\ C=1,\ \ v=1$





Heterogeneous Customer

To enable Paris Metro Pricing (PMP):

- How to determine the price of each subnetwork?
- How to assign resource to each subnetworks?

Heterogeneous Customer

- Leveraging Paris Metro Pricing (PMP)
  - Customer's utility:

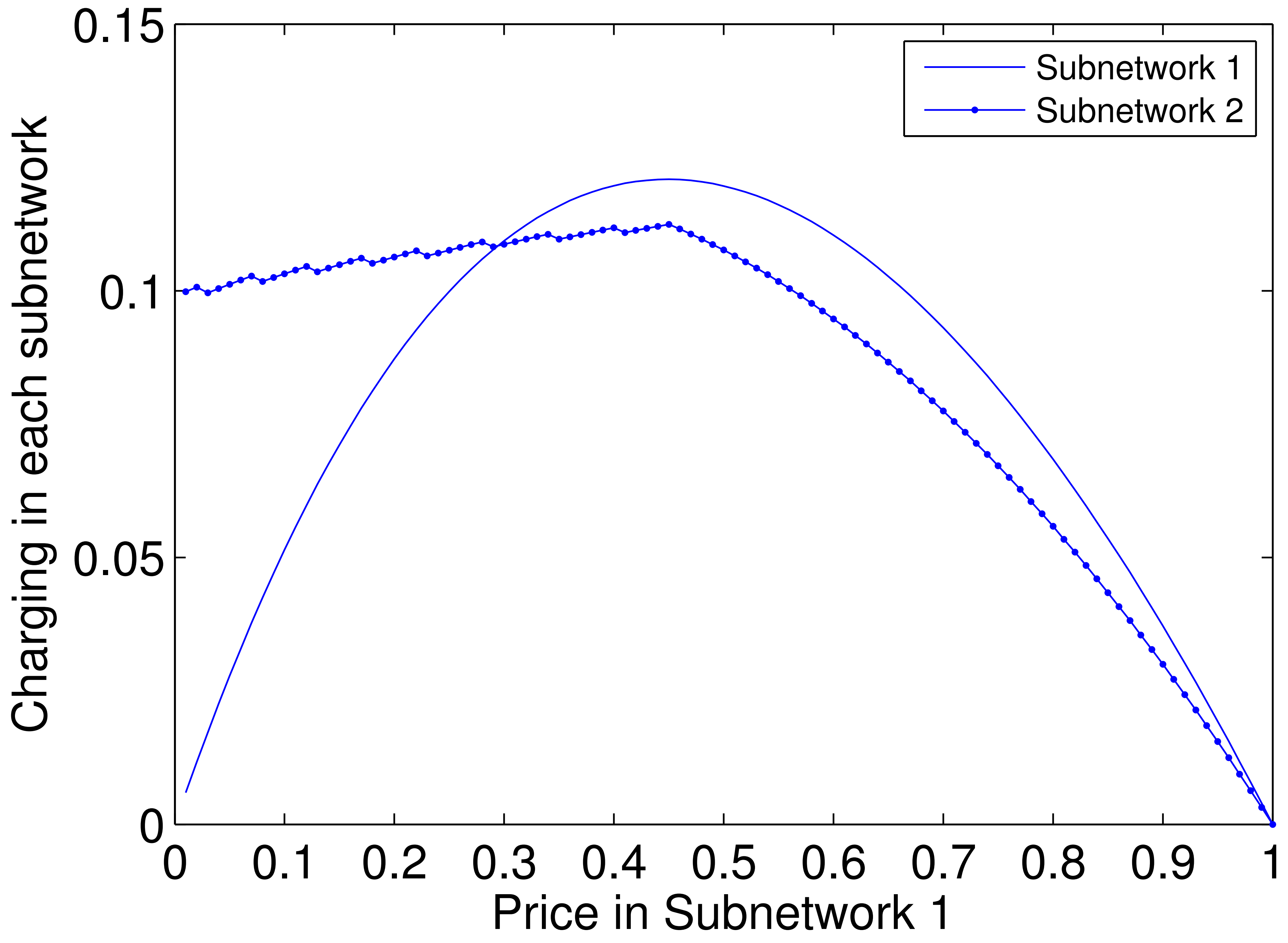
$$U(s_1,s_2,p_1,p_2) = D\int_{s_1}^{u^{-1}(p_1)} u(x)g(x)\,dx + D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} u(x)g(x)\,dx - p_1D\int_{s_1}^{u^{-1}(p_1)} q(x)\,dx - p_2D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)\,dx$$

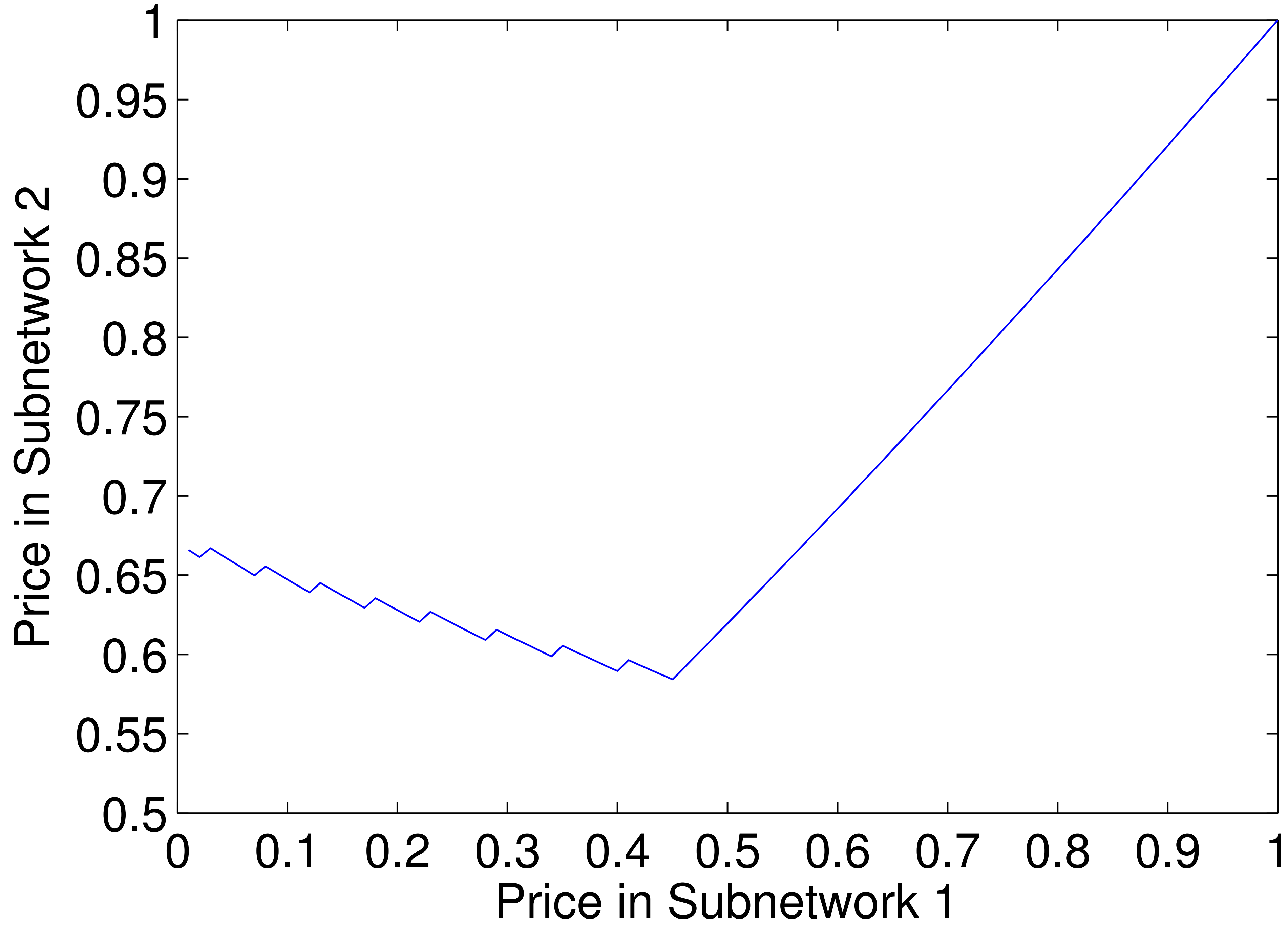
  - Operator's utility:

$$V(s_1,s_2,p_1,p_2) = p_1D\int_{s_1}^{u^{-1}(p_1)} q(x)\,dx + p_2D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)\,dx - vg\left(D\int_{s_1}^{u^{-1}(p_1)} q(x)\,dx + D\int_{s_2}^{\min(s_1,u^{-1}(p_2))} q(x)\,dx\right)$$

Heterogeneous Customer

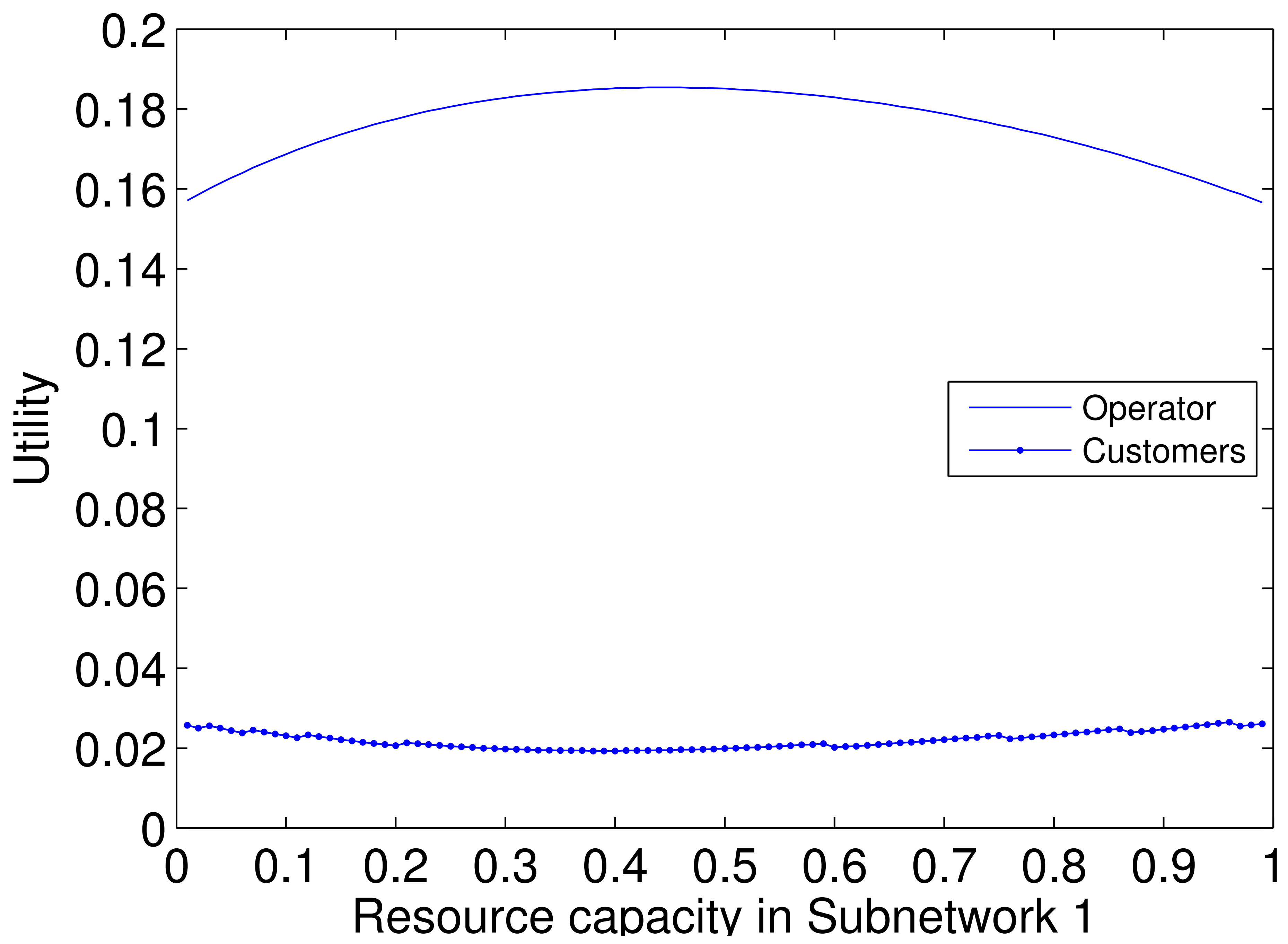
Example 5

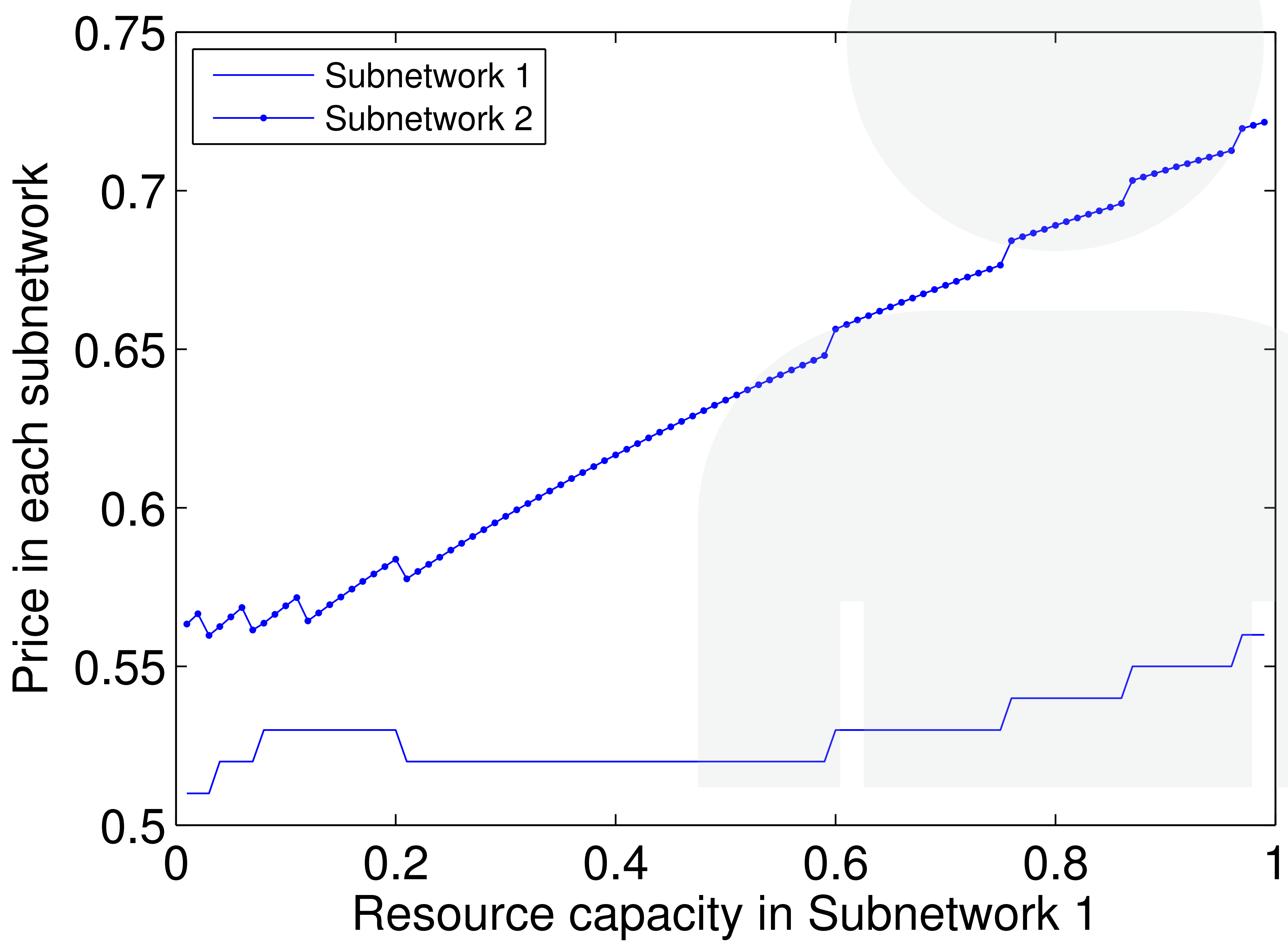




Heterogeneous Customer

Example 6





Conclusion

- Modelling the pricing problem in DCN as a Stackelberg game.
- Classify the market based on customers.
- For homogeneous and heterogeneous customers, Pareto-efficient solution at Stackelberg equilibrium.
- Introduce PMP scheme to heterogeneous customers case.

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Thank You